The fastest geometric optics primer in the Northeast

Zach Marin

February 20, 2021

1 Introduction

What follows is a brief introduction to optics using an approximation known as geometric optics. In this approximation, light travels in straight lines known as *rays*. This is useful for designing telescopes and microscopes, and this is the end goal we have in mind. Please note that this is not comprehensive, and optical elements such as apertures and stops, mirrors, prisms, wave plates, polarizers, the list goes on, are key to making good optical devices. This primer is a jumping off point to dive into the ocean of optics.

Much of this was built by following Optics by Eugene Hecht and some details were extracted from Principles of Optics by Born and Wolf. I've attempted to sprinkle in a few extra details I remember hearing from Dr. Joerg Bewersdorf, but any mistakes in this document are due to failures in my memory.

2 Lenses and refraction

A lens is a shaped piece of (usually) glass, plastic or metal that can disperse or focus light via refraction. *Refraction* is the change in direction of a (light) wave that passes from one medium (e.g. air) to another (e.g. glass). In other words: lenses bend light.

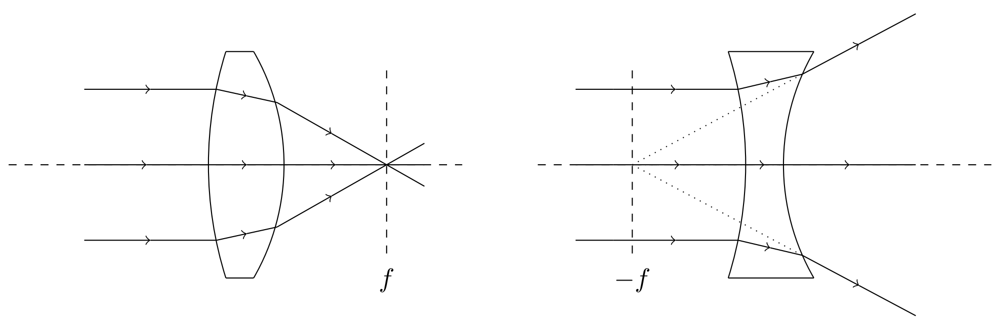


Figure 1: (a) A biconvex lens focusing light at focal length f. (b) A biconcave lens dispersing light coming from focal length $-f$.

When rays of light travel together in a straight line, such that all the rays are parallel, we say this light is *collimated*. Fig. 1a shows a biconvex lens bending collimated light toward a focal point f. The distance from the horizontal center of the lens to f is called the *focal length*, and is the most important characteristic of any lens.

Notice that we have drawn a dotted line through the vertical center of the lens and extending to "infinity" on either side of the horizontal center of the lens. This is called the *optical axis*. It is an imaginary line tracing the central optical ray of any system (e.g. a microscope system). The intersection of vertical and horizontal center of the lens is called the *optical center* of the lens.

For a biconcave lens, as shown in Fig. 1b, the focal length is the distance at which diverging rays would converge if they were traced backwards, as shown by the dotted lines.

Note that biconvex and biconcave are just two of many lens types. Lenses are classified by the curvature of their faces. If one side of a lens is convex and the other is flat, this is called planoconvex. If one side is concave and the other is flat, this is planoconcave. If one side is concave and the other is convex, this is called a meniscus lens. Think about how the ray diagrams would look for each of these cases.

Problem 1. Fig. 2 shows an eye with myopia (nearsightedness). The rays converge slightly in front of the retina (the retina is at the back of the eye), blurring objects that are far away. What kind of lens can we place in front of the eye to make the rays focus on the retina? Why does this work?

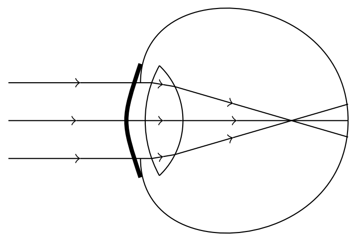


Figure 2: An eye with myopia.

2.1 Material

As mentioned above, refraction is caused by light moving from one material to another. Snell's law

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

describes light's change in direction as it passes between materials of different refractive indices. θ_0 and θ_1 are as shown in Fig. 3. n_0 and n_1 are the refractive indices of materials 0 and 1 (clear and gray, respectively, in Fig. 3). The *refractive index*, n_i, of a material i is a measure of the relative speed v_i at which light travels through material i as compared to the speed at which light travels in a vacuum, $c \approx 3 \times 10^8$ m/s. That is,

$$n_i = \frac{c}{v_i}.$$

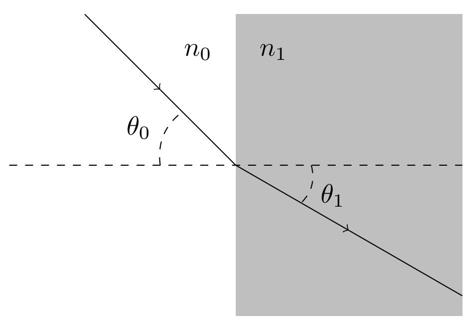


Figure 3: A ray of light passing from material 0 (e.g. air) to material 1 (e.g. glass) changes direction according to the angle of incidence θ_0 and the relative change in refractive indices n_0 to n_1.

Problem 2. Suppose we are choosing between lenses made out of two different types of glass. One is made of N-BK7, $n = 1.4990$, and the other is fused quartz, $n = 1.4567$. Assuming θ_0 and n_0 are constant, which lens will bend the light more (have a smaller θ_1)?

Problem 3. Suppose we are gluing two pieces of glass ($n = 1.51$) together to make a thicker piece of glass and we don't want the light to bend when it moves from one piece of glass to the other. Why is Canada Balsam a good choice of glue? What would the refractive index be for the best kind of glue?

2.2 Shape

In addition to refractive index, the shape of the lens helps determine lens focal length. Fig. 4 shows a biconvex lens carved from intersection of two spheres. This is closely related to the lens manufacturing process, where a diamond tool moving in a spherical arc carves out the shape of the lens from a *glass blank*, which is usually a disc or square piece of glass.

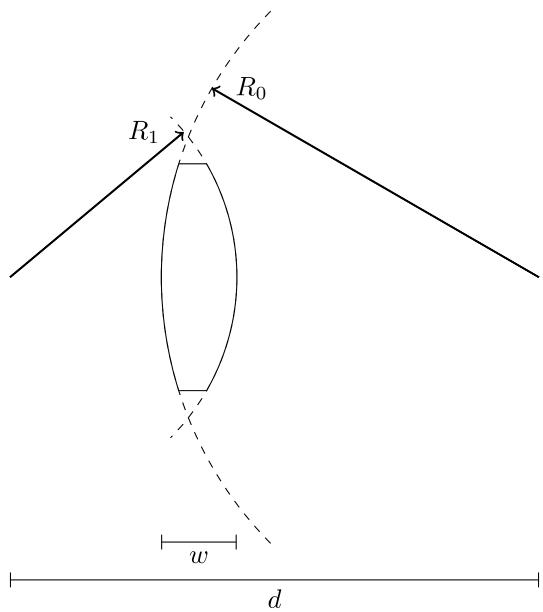


Figure 4: A biconvex lens of thickness w represented as the intersection of spheres of radius R_0 and R_1 separated by a distance d.

.

Putting this all together, the *lensmaker's equation*

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_0} - \frac{1}{R_1} + \frac{(n-1)w}{nR_0R_1}\right]$$

describes how the focal length of a lens is determined based on shape and refractive index of the lens n, assuming the lens is surrounded by $n_{\mathrm{air}} \approx 1.0003$. The equation is expressed in this way since $\frac{1}{f}$ is often referred to as the lens's *power*, that is the degree to which it can focus or diverge light. When w is small as compared to R_0 and R_1, we can use the *thin lens approximation* of the lensmaker's equation

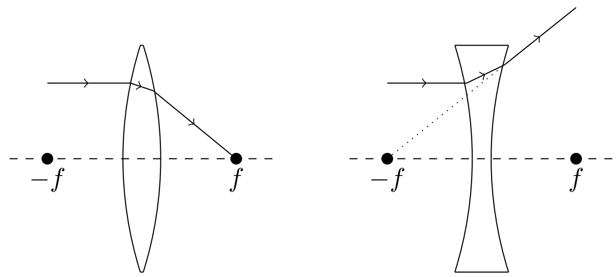
$$\frac{1}{f} \approx (n-1)\left[\frac{1}{R_0} - \frac{1}{R_1}\right].$$

Problem 4. Suppose we have a biconvex N-BK7 lens with $R_0 = 75$ mm, $R_1 = 200$ mm and $f = 240$ mm. What is the separation distance of the two spheres? Given the calculated lens thickness, what is the difference between the provided value for f and f in the thin-lens approximation? Does this make sense? Why or why not?

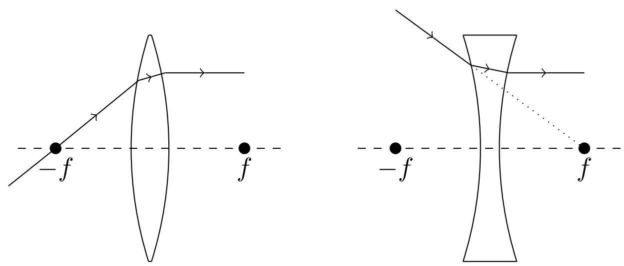
Problem 5. Draw the equivalent of Fig. 4 for a biconcave lens.

3 Rules for drawing ray diagrams

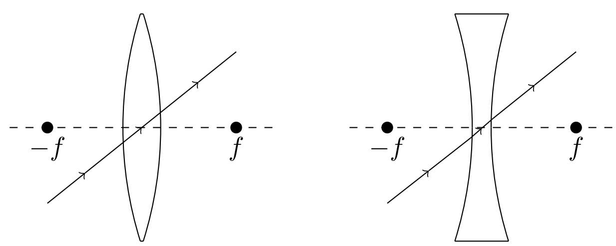
Rules 1-3 show how to draw rays passing through convex (left) and concave (right) lenses.



Rule 1: A ray traveling parallel to the optical axis will pass through the lens focal point.

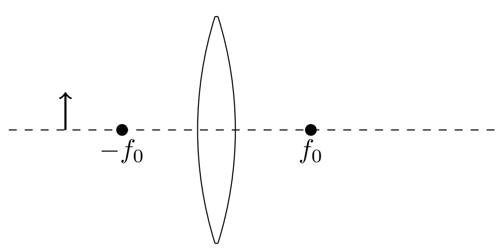


Rule 2: A ray traveling through the lens focal point will become parallel to the optic axis.

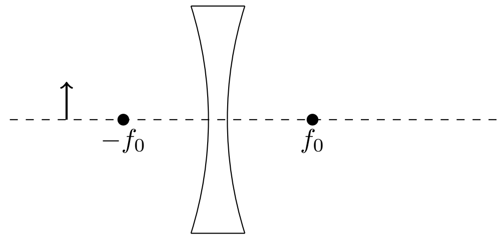


Rule 3: A ray passing through the lens optical center will continue in a straight line.

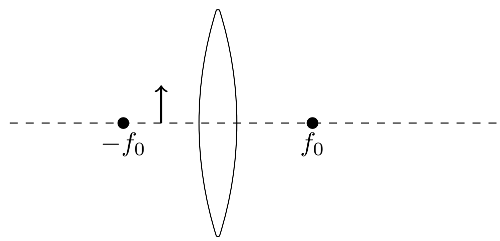
Problem 6. Below is a diagram for a simple microscope (à la Antonie van Leeuwenhoek) containing a single lens of focal length f_0. Draw a ray emitting parallel to the optical axis and heading right from the top of the arrow. Draw another ray heading toward $-f_0$ from the top of the arrow. Draw the rays until they intersect.



Problem 7. Draw the rays emitting parallel to the optical axis and toward $-f_0$ from the top of the arrow in the diagram below. Where is the image formed?



Problem 8. **An object that is too close to the lens.** Draw the rays emitting parallel to the optical axis and toward the optical center of the lens from the top of the arrow in the diagram below. Where is the image formed? Hint: how did we figure out the focal length of the biconcave lens in **Lenses and refraction**?



4 Magnification

The solution to problem 6 is shown in Fig. 5.

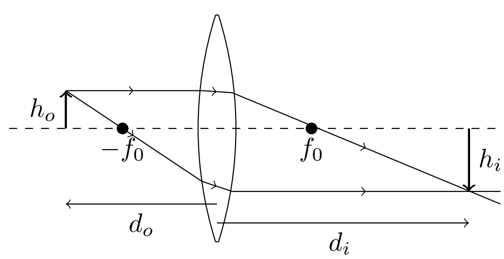
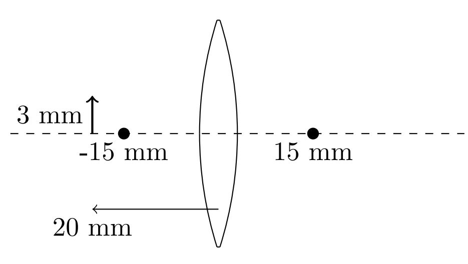


Figure 5: A labelled ray diagram for a simple microscope.

Here we have also drawn an arrow from the optical axis to the intersection of the rays. This arrow is the height h_i of the image of the object. The height of the object is given by h_o. The distance from the lens to the object is given by d_o and the distance from the lens to the image is given by d_i. The magnification of this lens is given by

$$M = \frac{h_i}{h_o} = \frac{-d_i}{d_o} = \frac{f}{f - d_o}. \tag{1}$$

Problem 9. Suppose we have the following single-lens system.



What is height of the image? What is the distance from the lens to the image?

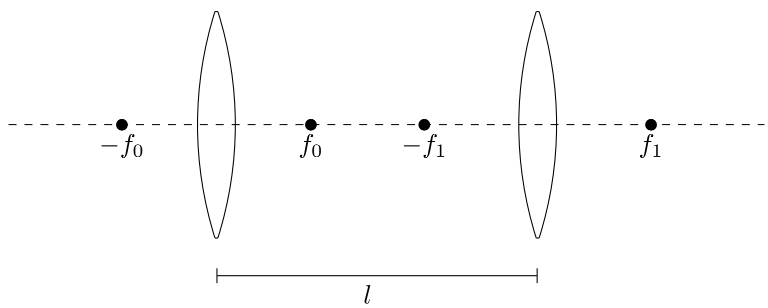
Problem 10. Suppose I grab a $f = 150$ mm focal length lens from an optics drawer in a lab. I find a 35 mm long paperclip at my desk. At what distance d_o from the lens will my paperclip be magnified $2\times$? What is the distance from the lens to the image?

5 Compound microscopes

It is difficult to achieve high magnification with a simple microscope such as in Problem 6 without distorting the image. As such, we most often build *compound microscopes*: microscopes containing two or more lenses.

5.1 Two lens systems

Below is a diagram of an optical system formed by two thin centered lenses.



For this system,

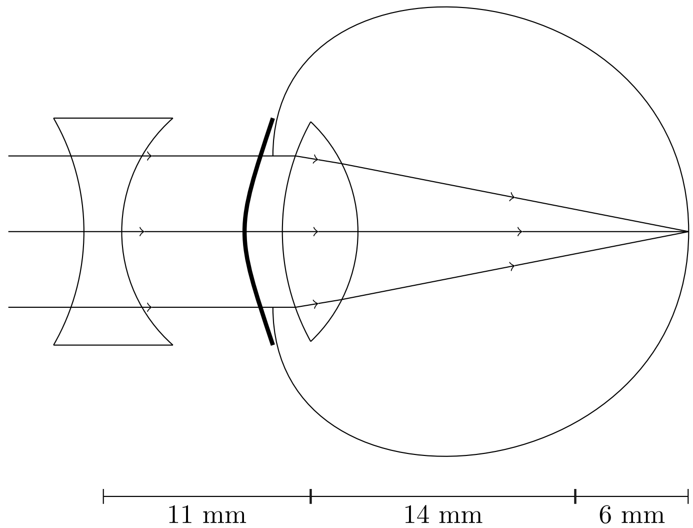
$$\frac{1}{f} = \frac{1}{f_0} + \frac{1}{f_1} - \frac{l}{f_0 f_1}. \tag{2}$$

The magnification of this system is given by

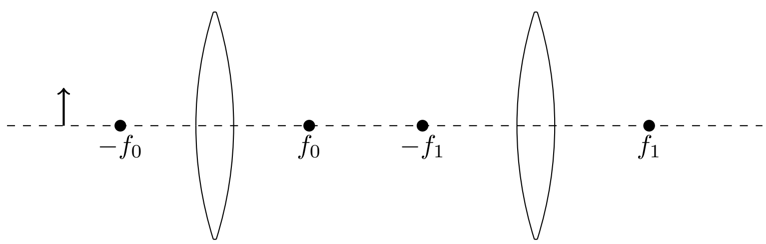
$$M = M_0 M_1$$

where M_0 is the magnification of the first lens and M_1 is the magnification of the second lens, each calculated as in Equation 1.

Problem 11. Recall that we fixed myopia in Problem 1. Now let's show how we fixed it in detail. Suppose we place a biconvex lens in front of an eye, as in the diagram below. What is the focal length of the biconcave lens? Note that glass prescription strength is equivalent to lens power. What is strength of this lens prescription? Is a person using this lens mildly or severely myopic?



Problem 12. Draw the rays emitting parallel to the optical axis and toward $-f_0$ from the top of the arrow in the diagram below. Where is the image formed?



5.1.1 4f systems

If we place two lenses with focal lengths f_0 and f_1 exactly $f_0 + f_1$ apart, we create a telescope called a $4f$ system (to understand why we call this $4f$, count the number of focal lengths: $-f_0$ to the first lens is 1, the lens to $f_0/-f_1$ is 2, $f_0/-f_1$ to the second lens is 3, and the second lens to f_1 is 4), shown in Fig. 6.

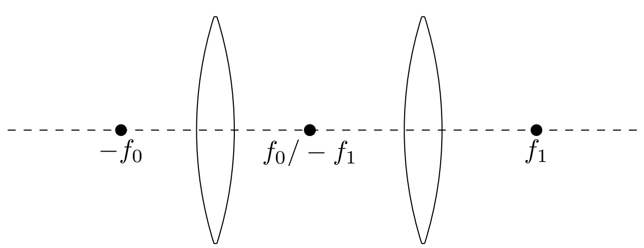
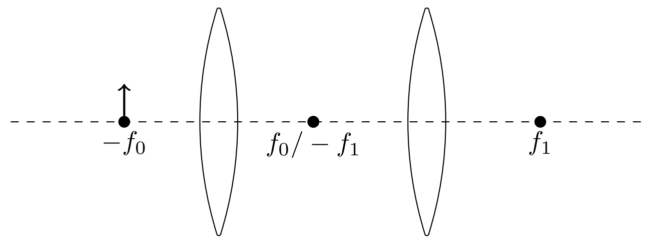


Figure 6: A $4f$ system.

$4f$ systems are fairly easy to work with, and as a result we like to use them when building microscopes. The problems below reveal some useful properties of $4f$ systems.

Problem 13. Show that the magnification of an object in focus in a $4f$ system is $M = \frac{-f_1}{f_0}$.

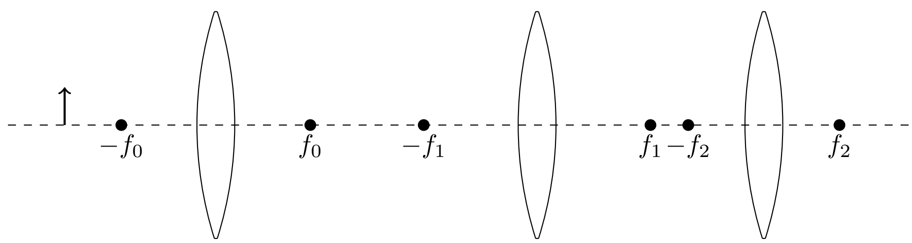
Problem 14. Draw the ray emitting parallel to the optical axis from the top of arrow in the diagram below. Where is the image formed? What are the focal lengths of this system?



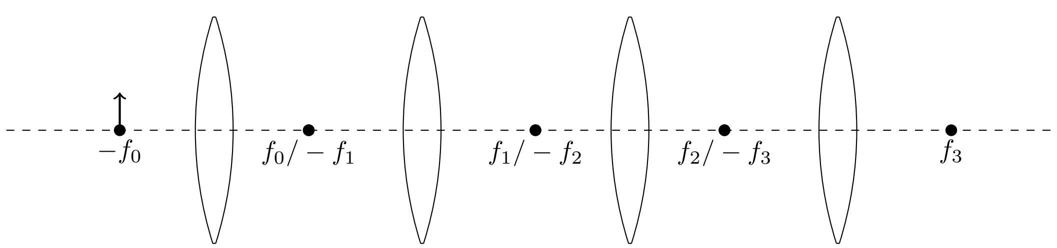
5.2 Multi-lens systems

This relation in Equation 2 is quite convenient: it means we can represent any two lenses with focal lengths f_0 and f_1 as a single lens of focal length f, as long as they are not in a $4f$ system. A $4f$ system maps an image at one focal length as an image at a different focal length. These observations allow us to recursively decompose a system of N lenses into sets of two lens.

Problem 15. Draw the rays emitting parallel to the optical axis and toward $-f_0$ from the top of the arrow in the diagram below. Where are the image planes? What is the focal length of this system? **Hint:** Once you've found the first image plane, start new rays from that image.



Problem 16. Draw the ray emitting parallel to the optical axis from the top of arrow in the diagram below. Where are the image planes? What are the focal lengths of this system? What is the magnification of this system?



5.3 Imaging onto a camera

We are interested in recording and analyzing the images created by our multi-lens systems. As such, we like to put cameras in image planes of interest (usually the final image plane).

Cameras record light in two-dimensional bins called pixels, converting a continuous image into a discrete set of pixels, as shown in Fig. 7. Here we are undersampling, as evidenced by the the coarseness of Fig. 7, right.

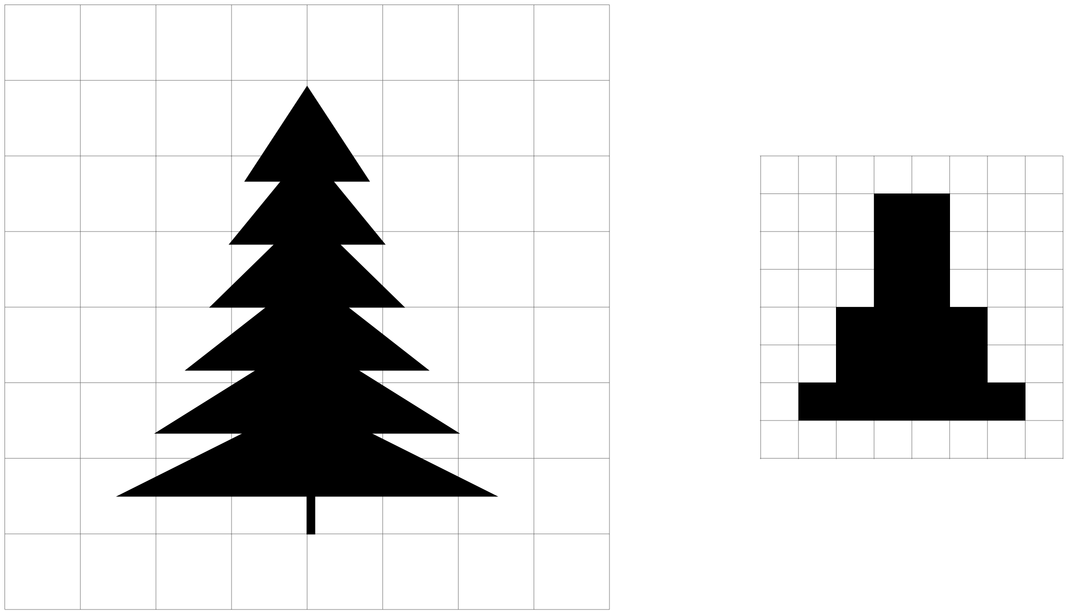


Figure 7: A camera chip forming a discrete image of a tree. Left: a tree projected onto a camera chip. Right: the resulting discrete image, black if a sufficient piece of tree is present within the pixel and white otherwise.

Ideally we will take an image with the best resolution possible. Our light has a resolution limit set by the diffraction limit of light, and this means our microscope point spread function is around 250 nm in diameter. According to Nyquist sampling, we need to make sure our camera samples at twice this spatial frequency–that is, at least 2 camera pixels for every 250 nm.

Problem 17. Suppose we are attempting to image an object onto a camera as below. The camera is 2048×2048 pixels and the pixel size is 6.45 μm $\times$ 6.45 μm. Suppose I want at least 3 pixels per point spread function. What magnification will we need to sample a 250 nm point spread function at this frequency? What is a set of focal lengths f_0, f_1 that will produce this magnification? Will the image of our object fit on the camera chip?